Precession Maneuver Performed by Applying a Uniform Train of Thrust Pulses

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A precession maneuver due to impulsive jets has been analyzed for the purpose of attitude control of an asymmetric spinning satellite. Basic assumptions of the analysis are that thrust pulses can be represented as a perfect square wave and that the axial asymmetry is considerably small. The Euler equation describing the precession maneuver has been solved by means of the Laplace transform and the perturbation method. Then a relation between the initial Euler phase angle and the rhumb angle has been found. Some analytical results are available for the preliminary estimation of fuel expenditure, spin-rate variation, and nutation angle resulting from the precession maneuver.

Introduction

THE attitude control is one of the basic problems for performing the mission of a spinning satellite. In particular the station acquisition maneuver for a geostationary satellite necessitates large attitude changes of more than 100°. Such an attitude change has commonly been effected by impulsive gas jets produced from an axial thruster. Various studies have been made to determine the timing of jet firings and to find the minimum fuel consumption and the minimum nutation.[1-4]

There are two methods useful to satellite builders concerned with the law of determining a phase angle which indicates the timing of jet firings. One is the rhumb line method, and the other is the great circle method. The great circle method requires continuous computation of the phase angle as a function of the attitude, while the rhumb line method requires only computation of a constant phase angle called the rhumb angle. This is the reason why the rhumb line method has been employed as the precession control law in spite of the fact that it is not so economical as the great circle method.

The rhumb line method has the significant feature that each pulse is uniformly spaced in the jet pulse train because of the constant phase angle. This feature makes it easy to express the jet pulse train in mathematical terms. However, the previous analyses have not yet provided an analytical solution of the Euler equation of an asymmetric spinning satellite subject to such a uniform train of jet pulses.

The analytical solution can facilitate the estimation of fuel expenditure, spin-rate variation, and nutation angle. The purpose of this paper is to find such a solution and to present some analytical expressions applicable to the estimation of fuel budget, the spin stabilization analysis, and the nutation damper design.

Solution of Euler Equation Describing Precession Maneuver

The jet pulses produced from an axial thruster in accordance with the rhumb line method are uniformly spaced at a rate of one per spin cycle. The motion of an asymmetric spinning satellite under the influence of such thrust pulses is governed by the Euler equation represented in the body coordinate system $0\text{-}x_1x_2x_3$. Here the origin 0 is the center of mass of the satellite and the axis x_1, x_2, x_3 are the principal axes of its inertia.

As it is well known, the thrust start-up and termination characteristics of realistic pulses are considerably different from those of ideal pulses. This fact makes it difficult to find an analytical solution of the equation of motion. For this reason, a basic assumption is made that thrust pulses can be represented as a perfect square wave characterized by the thrust level and the thrust pulsewidth.

With the condition that the axial thruster is fixed in parallel with the x_3-axis, the equation of motion is expressed as follows:

$$I_1\frac{d\dot{\Omega}_1}{dt}+(I_3-I_2)\dot{\Omega}_2\dot{\Omega}_3=F\ell\cos\beta f(t;t_s,t_a,N) \tag{1a}$$

$$I_2\frac{d\dot{\Omega}_2}{dt}+(I_1-I_3)\dot{\Omega}_3\dot{\Omega}_1=F\ell\sin\beta f(t;t_s,t_a,N) \tag{1b}$$

$$I_3\frac{d\dot{\Omega}_3}{dt}+(I_2-I_1)\dot{\Omega}_1\dot{\Omega}_2=0 \tag{1c}$$

where $\dot{\Omega}_i\,(i=1,2,3)$ is the x_i-component of the angular velocity, $I_i\,(i=1,2,3)$ is the principal moment of inertia about the x_i-axis, F is the thrust level, ℓ is the length of the moment arm, β is the angle between the x_1-axis and the direction of the precession torque, t is the time, t_s is the spin cycle period, t_a is the thrust pulsewidth, and N is the number of pulses. The function $f(t;t_s,t_a,N)$ represents a rectangular pulse train composed of N pulses; it is 1 for $nt_s\leq t<nt_s+t_a$ and 0 for $nt_s+t_a\leq t<(n+1)t_s$.

$$f(t;t_s,t_a,N)=\sum_{n=0}^{N-1}\{\eta(t-nt_s)-\eta(t-nt_s-t_a)\} \tag{2}$$

where $\eta(t)$ is a Heaviside unit step function which is 1 for $t\geq 0$ and 0 for $t<0$.

The mean of the transverse moments of inertia and their difference are introduced as follows:

$$I_0=(I_1+I_2)/2,\quad \Delta I=(I_1-I_2)/2 \tag{3}$$

The following dimensionless quantities

$$\kappa=I_3/I_0,\quad \lambda=\Delta I/I_0,\quad \alpha=t_a/t_s,\quad T=F\ell t_s^2/I_0 \tag{4}$$

$$\omega_i=\dot{\Omega}_it_s\,(i=1,2,3) \tag{5}$$

$$\tau=t/t_s \tag{6}$$

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are helpful to the subsequent analyses. Equation (1) is rewritten as follows by using Eqs. (3), (4), (5), and (6).

$$(1+\lambda)\frac{d\omega_1}{d\tau}+(\kappa-1+\lambda)\omega_3\omega_2=T\cos\beta E(\tau;\alpha,N) \quad (7a)$$

$$(1-\lambda)\frac{d\omega_2}{d\tau}-(\kappa-1-\lambda)\omega_3\omega_1=T\sin\beta E(\tau;\alpha,N) \quad (7b)$$

$$\kappa\frac{d\omega_3}{d\tau}=2\lambda\omega_1\omega_2 \quad (7c)$$

The function $E(\tau;\alpha,N)$ is related to Eq. (2) and defined as

$$E(\tau;\alpha,N)=\sum_{n=0}^{N-1}\{\eta(\tau-n)-\eta(\tau-nu-\alpha)\} \quad (8)$$

The following complex angular velocity is useful in the solution of Eq. (7).

$$\omega_\perp=\omega+j\omega_2 \quad (9)$$

Equation (7) is simplified as follows by using Eq. (9).

$$\frac{d\omega_\perp}{d\tau}+\lambda\frac{d\overline{\omega_\perp}}{d\tau}-j(\kappa-1)\omega_3\omega_\perp$$

$$+j\lambda\omega_3\overline{\omega_\perp}=Te^{j\beta}E(\tau;\alpha,N) \quad (10a)$$

$$\kappa\frac{d\omega_3}{d\tau}=\lambda\mathrm{Im}(\omega_\perp^2) \quad (10b)$$

where $\overline{\omega_\perp}$ is the complex conjugate of $\omega_\perp$.

Since the axial asymmetry λ is considerably small in many cases, the perturbation method based on the relation

$$\omega_\perp=\omega_\perp^{(0)}+\lambda\omega_\perp^{(1)},\qquad \omega_3=\omega_3^{(0)}+\lambda\omega_3^{(1)} \quad (11)$$

is instrumental in obtaining the analytical solution of Eq. (10). Substituting Eq. (11) into Eq. (10) and omitting the terms of λ^2, one has a zeroth-order equation and a first-order equation with respect to λ. The zeroth-order equation is expressed as

$$\frac{d\omega_\perp^{(0)}}{d\tau}-j(\kappa-1)\omega_3^{(0)}\omega_\perp^{(0)}=Te^{j\beta}E(\tau;\alpha,N) \quad (12a)$$

$$\frac{d\omega_3^{(0)}}{d\tau}=0 \quad (12b)$$

Then the first-order equation is expressed as

$$\frac{d\omega_\perp^{(1)}}{d\tau}+\frac{d\omega_\perp^{(0)}}{d\tau}-j(\kappa-1)(\omega_3^{(0)}\omega_\perp^{(1)}$$

$$+\omega_3^{(1)}\omega_\perp^{(0)})+j\omega_3^{(0)}\overline{\omega_\perp^{(0)}}=0 \quad (13a)$$

$$\kappa\frac{d\omega_3^{(1)}}{d\tau}=\mathrm{Im}(\omega_\perp^{(0)2}) \quad (13b)$$

The second expression, Eq. (12b), is integrated as

$$\omega_3^{(0)}=\omega_s=\pm 2\pi \quad (14)$$

where the plus indicates counterclockwise rotation while the minus indicates clockwise rotation. The first expression, Eq. (12a), is rewritten as follows by using Eq. (14).

$$\frac{d\omega_\perp^{(0)}}{d\tau}-j\mu\omega_\perp^{(0)}=Te^{j\beta}E(\tau;\alpha,N) \quad (15)$$

where μ is defined as

$$\mu=\omega_s(\kappa-1) \quad (16)$$

The first expression, Eq. (13a) is approximated as

$$\frac{d\omega_\perp^{(1)}}{d\tau}-j\mu\omega_\perp^{(1)}=-\left(\overline{\frac{d\omega_\perp^{(0)}}{d\tau}}+j\omega_s\overline{\omega_\perp^{(0)}}\right) \quad (17)$$

The following relation is valid because $\dot{\Omega}_1=\dot{\Omega}_2=0$ at $t=0$.

$$\omega_\perp^{(0)}(0)=\omega_\perp^{(1)}(0)=0 \quad (18)$$

Applying the Laplace transform to Eqs. (15) and (17) and considering Eq. (18), one has

$$\mathcal{L}\{\omega_\perp^{(0)}\}=\frac{T}{\mu}e^{j\beta}\left\{\frac{\mu}{S^2+\mu^2}+j\left(\frac{1}{S}-\frac{S}{S^2+\mu^2}\right)\right\}\frac{(1-e^{-\alpha S})(1-e^{-NS})}{1-e^{-S}} \quad (19)$$

$$\mathcal{L}\{\omega_\perp^{(1)}\}=-\frac{T}{\mu}e^{-j\beta}\left\{\frac{\mu}{S^2+\mu^2}+j\frac{\omega_s}{\mu}\left(\frac{1}{S}-\frac{S}{S^2+\mu^2}\right)\right\}\frac{(1-e^{-\alpha S})(1-e^{-NS})}{1-e^{-S}} \quad (20)$$

where S is a Laplace parameter. The following formulas are available to the Laplace transform inversion of Eqs. (19) and (20).

$$\mathcal{L}^{-1}\left\{\frac{(1-e^{-\alpha S})(1-e^{-NS})}{S(1-e^{-S})}\right\}=E(\tau;\alpha,N) \quad (21)$$

$$\mathcal{L}^{-1}\left\{\frac{S(1-e^{\alpha S})(1-e^{-NS})}{(S^2+\mu^2)(1-e^{-S})}\right\}=\sum_{n=0}^{N-1}\{\cos\mu(\tau-n)\eta(\tau-n)$$

$$-\cos\mu(\tau-n-\alpha)\eta(\tau-n-\alpha)\}\equiv C(\tau;\mu,\alpha,N) \quad (22)$$

$$\mathcal{L}^{-1}\left\{\frac{\mu(1-e^{-\alpha S})(1-e^{-NS})}{(S^2+\mu^2)(1-e^{-S})}\right\}=\sum_{n=0}^{N-1}\{\sin\mu(\tau-n)\eta(\tau-n)$$

$$-\sin\mu(\tau-n-\alpha)\eta(\tau-n-\alpha)\}\equiv S(\tau;\mu,\alpha,N) \quad (23)$$

The solution $\omega_\perp$ is expressed as

$$\omega_\perp=\frac{T}{\mu}\Big[(e^{j\beta}-\lambda e^{-j\beta})S(\tau;\mu,\alpha,N)-j\left(e^{j\beta}-\frac{\lambda}{\kappa-1}e^{-j\beta}\right)$$

$$\{C(\tau;\mu,\alpha,N)-E(\tau;\alpha,N)\}\Big] \quad (24)$$

Fuel Expenditure, Spin-Rate Variation and Nutation Angle

The Euler angles θ and ψ are convenient in representing the attitude of a spinning satellite in the inertia coordinate system $0-XYZ$ where the Z-axis lies in the direction of the north pole and the X-axis lies in the direction of the vernal equinox. The angle θ denotes the spin axis inclination, and the angle ψ denotes the spin equator right ascension. The Euler phase angle φ expressing the spinning motion is defined as the angle between the x_1-axis and the direction of the ascending node of the spin equator. Figure 1 shows schematically the relationships of the angles θ, ψ, and φ. The relation between $\dot{\theta}$, $\dot{\psi}$, $\dot{\varphi}$

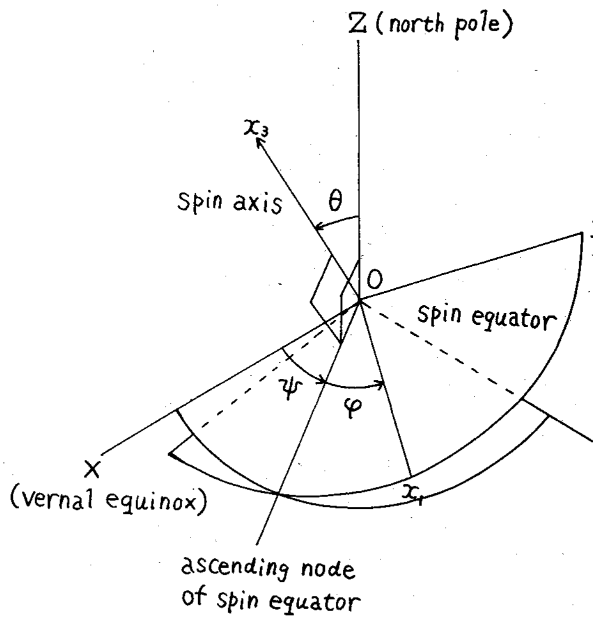

Fig. 1 Spin axis geometry.

and $\dot{\Omega}_1$, $\dot{\Omega}_2$, $\dot{\Omega}_3$ is given by

$$\dot{\theta}=\dot{\Omega}_1\cos\varphi-\dot{\Omega}_2\sin\varphi \tag{25a}$$

$$\dot{\varphi}=\dot{\Omega}_3-\dot{\psi}\cos\theta \tag{25b}$$

Equation (25) is rewritten as

$$\frac{d\theta}{d\tau}=\mathrm{Re}(\omega_\perp e^{j\varphi}) \tag{26a}$$

$$\frac{d\varphi}{d\tau}=\omega_3-\frac{d\psi}{d\tau}\cos\theta \tag{26b}$$

The second expression, Eq. (26b), can be integrated as follows because $\omega_3 \approx \omega_s \gg (d\psi/d\tau)$.

$$\varphi=\varphi_0+\omega_s\tau \tag{27}$$

where φ_0 is the initial Euler phase angle indicating the beginning of the precession maneuver.

The spin axis inclination change $\Delta\theta$ caused by the jet pulse train is given by the following integral derived from the first expressions, Eq. (26a), and Eq. (27).

$$\Delta\theta=\mathrm{Re}\left(e^{j\varphi_0}\int_0^N\omega_\perp e^{j\omega_s\tau}d\tau\right) \tag{28}$$

Substituting Eq. (24) into Eq. (28) and omitting the terms independent of N, one has

$$\frac{\Delta\theta}{N}=-\frac{2T}{\omega_s^2\mathrm{K}}\sin\frac{\omega_s\alpha}{2}\sin\left(\varphi_0+\beta+\frac{\omega_s\alpha}{2}\right) \tag{29}$$

The previous analyses found the rhumb angle ϕ and the number of pulses N for the precession maneuver required to change the initial attitude defined as θ_1, ψ_1 into the final attitude defined as θ_2, ψ_2. They are given by

$$\cot\phi=\frac{1}{\Delta\psi}\log\left(\frac{\tan\frac{\theta_2}{2}}{\tan\frac{\theta_1}{2}}\right) \tag{30}$$

$$N=\frac{I_3\dot{\Omega}_3^2}{2F\ell}\left|\frac{\Delta\theta}{\sin\frac{\dot{\Omega}_3t_a}{2}\cos\phi}\right| \tag{31}$$

where $\Delta\theta=\theta_2-\theta_1$ and $\Delta\psi=\psi_2-\psi_1$.

If the angle φ_0 satisfies the relation

$$\varphi_0+\beta+\frac{\omega_s\alpha}{2}=\phi\pm\frac{\pi}{2} \tag{32}$$

then Eq. (29) is reduced to

$$\frac{\Delta\theta}{N}=\pm\frac{2T}{\omega_s^2\kappa}\sin\frac{\omega_s\alpha}{2}\cos\phi \tag{33}$$

Since Eq. (33) is equivalent to Eq. (31), Eq. (32) expresses a conversion formula between the initial Euler phase angle and the rhumb angle.

The fuel expenditure δm per one jet pulse is dependent on the specific impulse I_{sp} of the fuel, and expressed as

$$\delta m=\frac{Ft_a}{I_{sp}g} \tag{34}$$

where g is the gravitational acceleration. The fuel expenditure Δm required for the previous attitude change is easily derived from Eqs. (31) and (34).

$$\Delta m=N\delta m=\frac{I_3\omega_s^2\alpha}{2I_{sp}g\ell t_s}\frac{\Delta P}{\left|\sin\frac{\omega_s\alpha}{2}\right|} \tag{35}$$

where ΔP is the precession angle defined as

$$\Delta P=\left|\frac{\Delta\theta}{\cos\phi}\right| \tag{36}$$

Equation (35) shows that the fuel consumption is proportional to ΔP. Equation (35) also shows that more fuel is expended as I_3 and α become larger.

From the third expression, Eq. (1c) or Eq. (7c), it is apparent that changes in spin usually occur along with precession. The spin-rate variation $\Delta\dot{\Omega}_3$ due to the precession maneuver is given by the following integral derived from the second expression. Eq. (13b).

$$\Delta\dot{\Omega}_3=\frac{\lambda\Delta\omega_3^{(1)}}{t_s}=\frac{\lambda}{t_s\kappa}\mathrm{Im}\left(\int_0^N\omega_\perp^{(0)2}d\tau\right) \tag{37}$$

Substituting Eq. (24) into Eq. (37) and omitting the terms independent of N, one has the spin-rate variation per one jet pulse.

$$\frac{\Delta\dot{\Omega}_3}{N}=\frac{2\lambda}{t_s\kappa\mu}\left(\frac{T}{\mu}\right)^2\left\{\sin\frac{\mu\alpha}{2}\sin\frac{\mu(1-\alpha)}{2}\Big/\sin\frac{\mu}{2}-\frac{\mu\alpha}{2}\right\}\sin2\beta \tag{38}$$

Equation (38) shows that the precession maneuver does not cause changes in spin if the satellite is symmetric or if the axial thruster is fixed on the x_1- or x_2-axis, that is, $\lambda=0$ or $\beta=0°$, 90°, 180°, 270°. This consequence is easily understood from the facts that the second expression, Eq. (10b), is in proportion to λ and that Eq. (24) is linearly dependent on $e^{\pm j\beta}$. Equation (38) also shows that the amount of change in spin takes a maximum value at $\beta=45°$, 135°, 225°, 315°. From Eq. (38), one finds that the sign of $\lambda\sin2\beta$ influences which sign occurs in spin-up or spin-down. A further remark obtained from Eq. (38) is that an increase in I_0 and a decrease in λ reduce the change in spin.

The complex angular velocity $\omega_{\perp N}$ for $\tau > N$ is significant in finding the nutation angle resulting from the precession maneuver. Equation (24) for $\tau > N$ is reduced to

$$\omega_{\perp N} = \frac{2Tk_N}{\mu}\left[e^{j\beta + j\mu(\tau - \tau_N)} - \lambda e^{-j\beta}\left\{\cos\mu(\tau-\tau_N) + \frac{j}{\kappa - 1}\sin\mu(\tau - \tau_N)\right\}\right] \tag{39}$$

where k_N and τ_N are defined as

$$k_N = \sin\frac{\mu\alpha}{2}\sin\frac{\mu N}{2}\Big/\sin\frac{\mu}{2} \tag{40}$$

$$\tau_N = \frac{N - 1 + \alpha}{2} \tag{41}$$

The angular momentum of the satellite is composed of the component M_3 parallel to the spin axis and the complex component $M_\perp$ perpendicular to it. They are expressed as

$$M_3 = I_3\dot{\Omega}_3 = \frac{I_3}{t_s}\omega_s \tag{42a}$$

$$M_\perp = I_1\dot{\Omega}_1 + jI_2\dot{\Omega}_2 = \frac{I_0}{t_s}(\omega_{\perp N} + \lambda\overline{\omega}_{\perp N}) \tag{42b}$$

Substituting Eq. (39) into the second expression, Eq. (42b), and omitting the terms of λ^2, one has

$$M_\perp = \frac{I_0}{t_s}\frac{2Tk_N}{\mu}e^{j\beta}\left\{\cos\mu(\tau-\tau_N) - \frac{\lambda\kappa}{\kappa-1}\sin 2\beta\,\sin\mu(\tau-\tau_N) + j\left(1 - \frac{\lambda\kappa}{\kappa - 1}\cos 2\beta\right)\sin\mu(\tau-\tau_N)\right\} \tag{43}$$

The nutation angle θ_N is given by

$$\theta_N \approx \tan\theta_N = \frac{(\langle |M_\perp|^2\rangle)^{1/2}}{|M_3|} = \frac{2T|k_N|}{\kappa\mu\omega_s}\left[1 + \frac{1}{2}\left(\frac{\lambda\kappa}{\kappa-1}\right)^2 - \frac{\lambda\kappa}{\kappa-1}\cos 2\beta\right]^{1/2} \tag{44}$$

where $(\langle |M_\perp|^2\rangle)^{1/2}$ is the square root of the average of $|M_\perp|^2$ per one spin cycle.

From Eqs. (40) and (44), one finds the maximum nutation angle

$$\theta_{N\max} = \frac{2T}{\kappa\mu\omega_s}\left(\sin\frac{\mu\alpha}{2}\Big/\sin\frac{\mu}{2}\right)\left[1 + \frac{1}{2}\left(\frac{\lambda\kappa}{\kappa-1}\right)^2 - \frac{\lambda\kappa}{\kappa-1}\cos 2\beta\right]^{1/2} \tag{45}$$

Table 1 Dynamic characteristics of the satellite

Initial spin rate		100 rpm
Thrust level		22 N
Length of moment arm		0.65 m
Specific impulse of hydrazine		210 sec
Moment of inertia before apogee motor burn	I_1	28.5 kg·m^2
	I_2	31.5 kg·m^2
	I_3	36.0 kg·m^2
Moment of inertia after apogee motor burn	I_1	24.5 kg·m^2
	I_2	27.5 kg·m^2
	I_3	31.0 kg·m^2

Equation (45) shows that $\theta_{N\max}$ decreases if $\lambda\cos 2\beta$ is positive and increases if it is negative. The axial thruster should therefore be positioned in consideration of which of I_1 and I_2 is the larger. Equation (45) also shows that an increase in I_3 and a decrease in α help in the reduction of nutation. If the nutation damps exponentially, the following expression gives the time constant t_D necessary to a nutation damper applied to reduce $\theta_{N\max}$.

$$t_D = t_R/\log(\theta_{N\max}/\theta_R) \tag{46}$$

where θ_R is the residual nutation angle and t_R is the allowable time which does not put any inconvenience for the mission of the satellite.

Numerical Results and Discussions

In general, two gross precession maneuvers are indispensable to perform the mission of a geostationary satellite. The first is called reorientation, which changes the transfer orbit injection attitude into the apogee motor burn attitude. The other is called normalization, which directs the spin axis towards the north pole after the apogee motor burn.

Table 1 shows the dynamic characteristics of a geostationary satellite presented for discussions. From Table 1, one finds that the mass property of this satellite is expressed as $I_0 = 30$ kg·m^2, $\kappa = 1.2$, and $\lambda = -0.05$ in reorientation while $I_0 = 26$ kg·m^2, $\kappa \approx 1.2$, and $\lambda \approx -0.06$ in normalization. Table 1 indicates that this satellite has an axial thruster characterized by $F = 22$ N, $\ell = 0.65$ m, and $I_{sp} = 210$ sec. Then the spin cycle is expressed as $t_s = 600$ msec because the initial spin rate is 100 rpm.

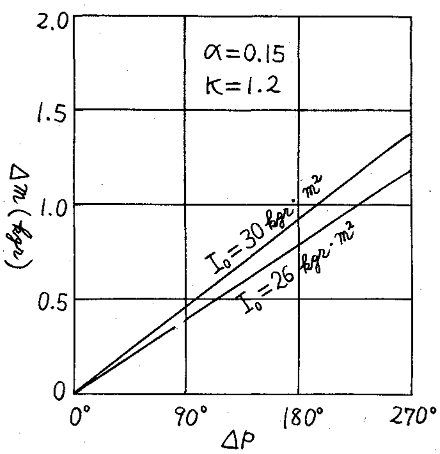

Fig. 2 Fuel expenditure.

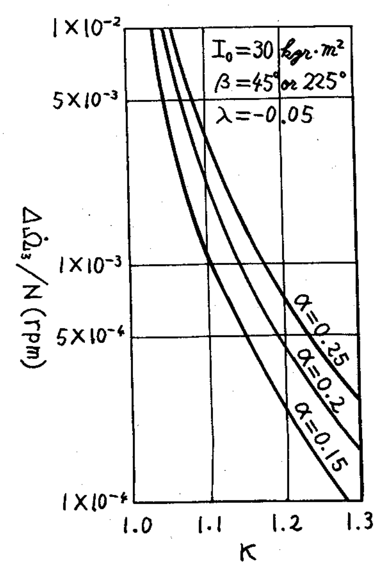

Fig. 3 Spin-rate variation.

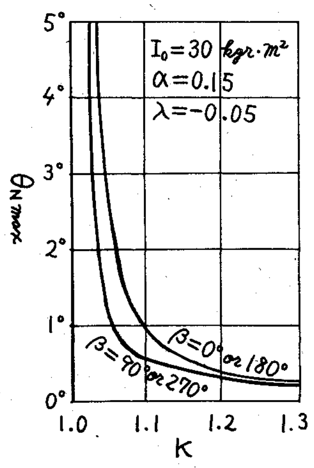

Fig. 4 Maximum nutation angle.

Figure 2 shows the fuel consumption necessary to the attitude change specified by the precession angle on condition that $\alpha = 0.15$, that is, $t_a = 90$ msec. Since the precession angle is estimated at about 180° in reorientation and about 130° in normalization, the fuel consumption predicted from Fig. 1 is about 0.91 kg in reorientation and about 0.56 kg in normalization. The fuel budget required for both precession maneuvers is therefore estimated at about 1.47 kg.

Figure 3 shows the spin-rate variation which resulted from reorientation on condition that $\beta = 45°$ or 225°. From Fig. 3, one finds that changes in spin caused by reorientation of the satellite specified as $\kappa = 1.2$ and $\alpha = 0.15$ are less than 0.3 rpm because the number of pulses necessary to the reorientation is about 960. Apparent features of Fig. 3 are that changes in spin vary almost exponentially as κ varies and that they decrease as α becomes smaller.

From Eq. (45), it is evident that if λ is negative, then $\theta_{N\max}$ takes a maximum at $\beta = 0°$ or 180° and a minimum at $\beta = 90°$ or 270°. Figure 4 shows a maximum and a minimum of the maximum nutation angle which resulted from reorientation. Figure 4 illustrates a tendency for $\theta_{N\max}$ to increase rapidly when κ becomes less than 1.05. From Fig. 4, one finds that a maximum of $\theta_{N\max}$ is estimated at about 0.39° in reorientation of the satellite specified as $\kappa = 1.2$ and $\alpha = 0.15$. Therefore, the time constant of a nutation damper should be less than 5.3 min in order to diminish the maximum nutation. Here the assumption is made that $\theta_R = 0.023°$ and $t_R = 15$ min in Eq. (46).

Concluding Remarks

A primary result of the analysis is an analytical solution of the Euler equation describing a precession maneuver due to a uniform train of rectangular thrust pulses. The Laplace transform inversion formulas expressed as Eqs. (21), (22), and (23) have been instrumental in analyzing such a pulse train. Then the perturbation method based on Eq. (11) was employed as a method of solution for taking an axial asymmetry into account.

Significant results derived from the solution are the analytical expressions of fuel expenditure, spin-rate variation, and maximum nutation angle. Numerical results for a geostationary satellite show that they are applicable to the preliminary estimation of fuel budget required for precession maneuvers, the spin stabilization analysis, and the nutation damper design. Then the conversion formula between the initial Euler phase angle and the rhumb angle was developed.

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